

Understanding the movements of metal whiskers

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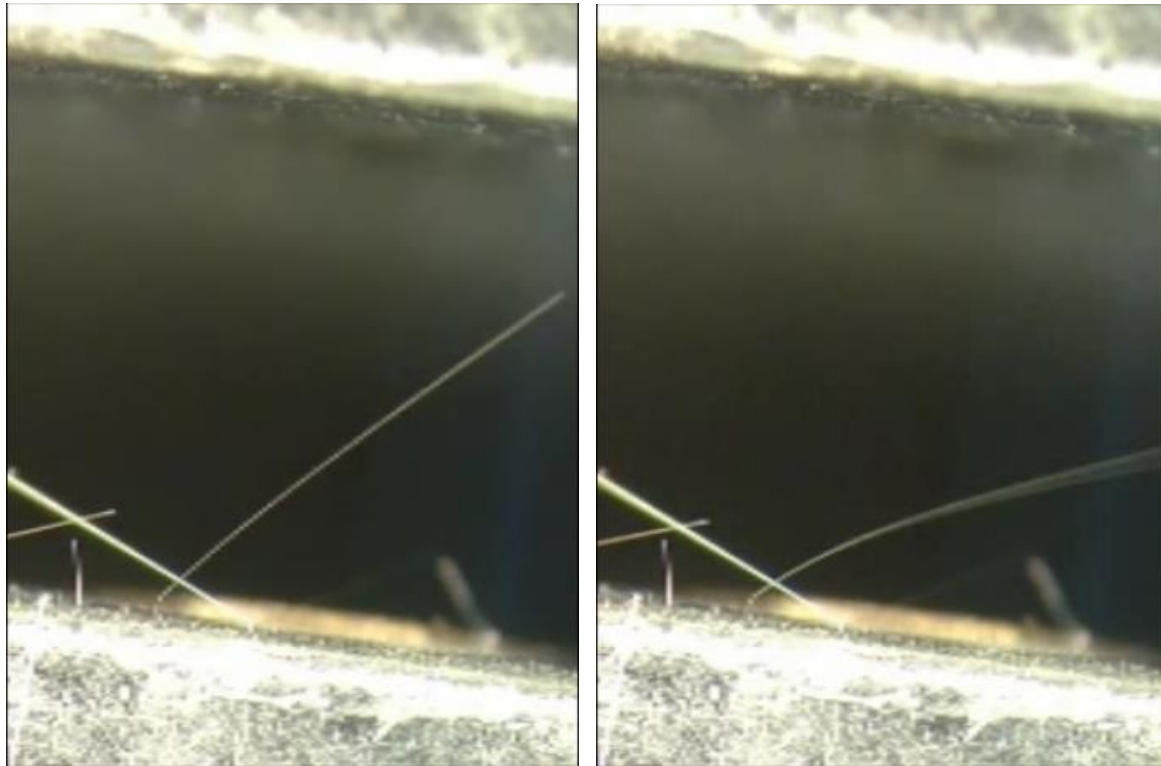
Outline

- Motivation
- Approach
- Whisker mechanics in elastic beam model
- 5 potential mechanisms of movements
 - Air flow effects
 - Brownian movements
 - Mechanical vibrations
 - Garden hose instability
 - Electric instabilities: ionic diffusion and nonequilibrium charges
- Conclusions

Motivation: observations by many

- Examples of movies
 - <https://nepp.nasa.gov/whisker/video/index.html>
 - <https://www.youtube.com/watch?v=HbWRPAVzdmc>
- Movies imply whisker movement due to minute air flows (expiration towards whiskers, etc.;), thanks Jay B for explanations
- Anecdotal evidence by many: whiskers move spontaneously, without any intended stimuli (thanks to the discussions during one of the recent teleconferences).
- I saw spontaneous movements myself with a flashlight and a magnifying glass on huge zinc whiskers (thanks Jay B)
- But was it spontaneous? Could it be documented?
- Did others see spontaneous whisker movements?
- **My contribution here: what is possible theoretically?**

Note:
these are high amplitude movements

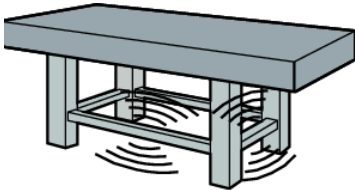


- with characteristic times of 0.1 – 0.01 s
- often, nearest neighbors move incoherently

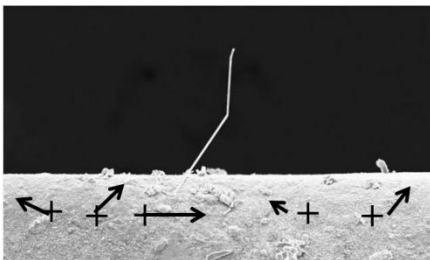
Proposed mechanisms of metal whisker movements



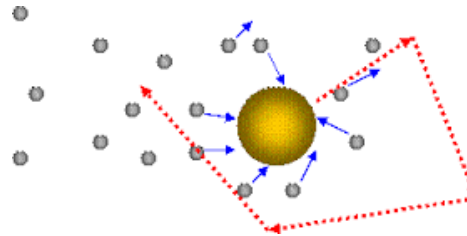
Air flow (NASA team)



External mechanical vibrations (B. Rollins)



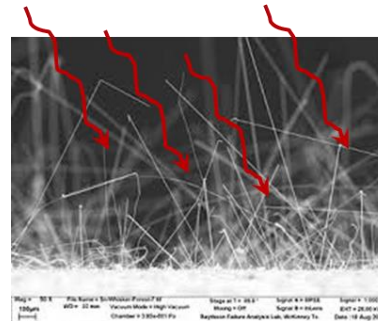
Random electric fields due to ionic diffusion (VK)



Brownian movements (G. Davy)



Garden hose instability (VK)



Random electric fields due to light induced recharging (VK)

Announcing final results (in case you do not have time)

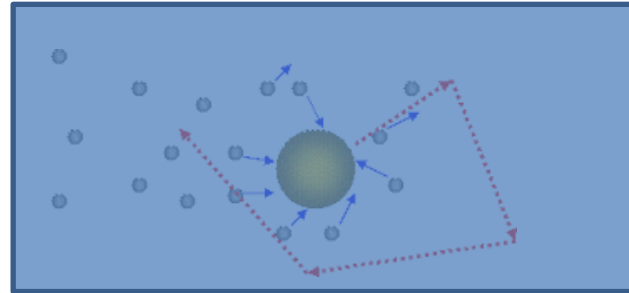
Certainly possible



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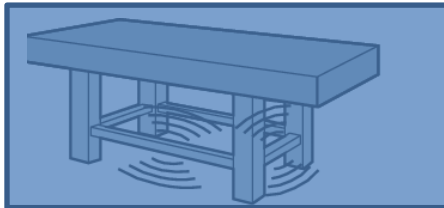
Air flow (NASA team)

unlikely



Brownian movements (G. Davy)

unlikely



External mechanical vibrations (B. Rollins)

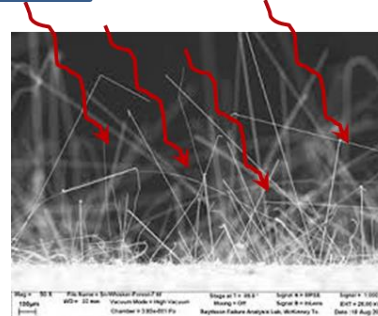
impossible



Random electric fields due to ionic diffusion (VK)

impossible

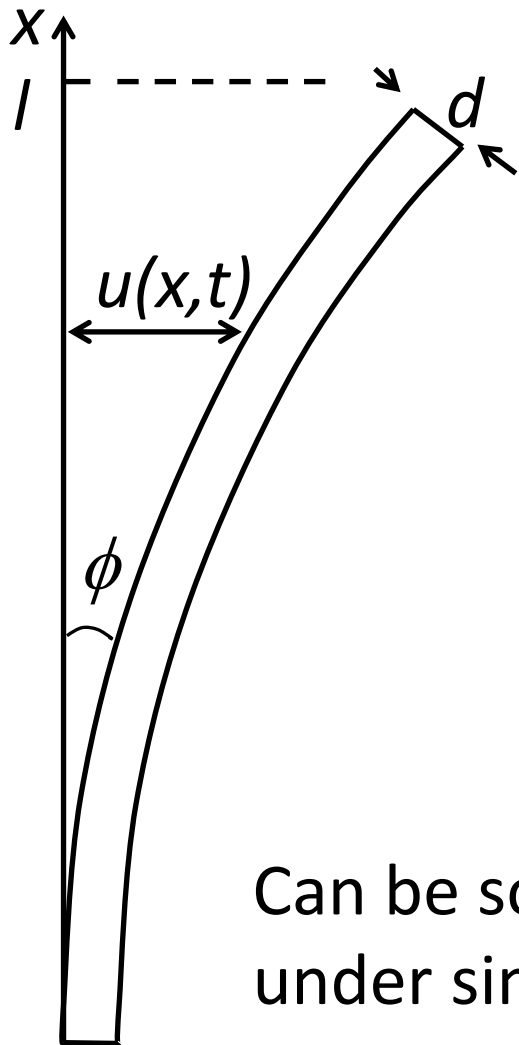
Garden hose instability (VK)



plausible

Random electric fields due to light induced recharging (VK)

Elastic beam model for whiskers



Movements reversible – elasticity
Euler-Bernoulli equation

$$YI \frac{\partial^4 u}{\partial x^4} + \rho A \frac{\partial^2 u}{\partial t^2} + B \frac{\partial u}{\partial t} = f(x, t)$$

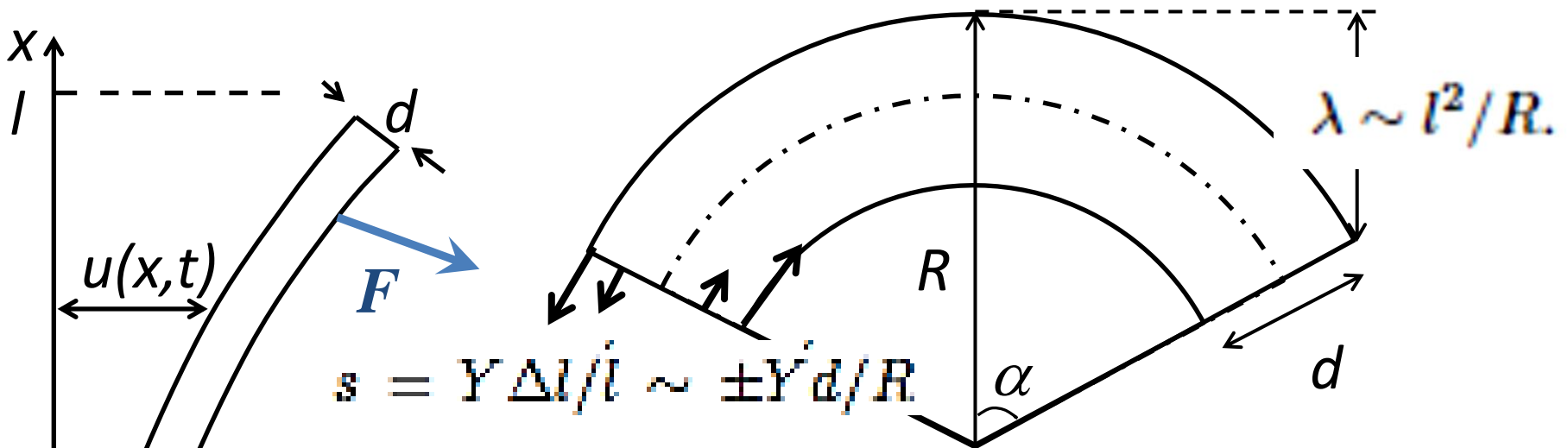
u – deflection, t – time, Y -Young's moduli, I -area moment of inertia, ρ -material density, A -area, B -friction coefficient, f -force at point x on the beam

Can be solved exactly for cantilever beams under simple forces; otherwise -- useless

Order-of-magnitude approximations

- Phenomena are versatile and parameters unknown → approximations necessary, exact models excessive
- Neglect all numerical multipliers
 - Example: area of a circle $\sim d^2$ instead of $\pi(d/2)^2$
- Including even integrals and derivatives
 - Example: $\int_0^1 x^2 dx \sim \langle x^2 \rangle \cdot \Delta x = \left(\frac{1}{2}\right)^2 \cdot 1 = \frac{1}{4}$ instead of $\frac{1}{3}$
 $\frac{d(x^2)}{dx} \sim \frac{x^2}{x} = x$ instead of $2x$
- It works! Errors mostly cancel each other to the accuracy of insignificant multipliers similar to cancelations of random displacements in diffusion
- = ‘Errors propagate by diffusion’ (attributed to Einstein)
- It is efficient, fast, inexpensive... and almost forgotten

Basics of whisker bending



Torque sAd by stresses \sim Torque Fl by force F


$$\lambda \sim \frac{Fl^3}{Yd^4} \sim \frac{F}{K} \left(\frac{l}{d} \right)^2 \equiv \frac{F}{K_b}$$

K is compressive spring constant,
 $K_b = K(d/l)^2$
 is bending spring constant


- Major equation: deflection vs. force/geometry
- Bending is easier than compressing
- Very sensitive to whisker geometry

Useful consequences

Frequency of
harmonic oscillator


$$\omega_b \sim \sqrt{\frac{K_b}{m}} \sim \frac{d}{l^2} \sqrt{\frac{Y}{\rho}} \sim \frac{d}{l^2} v_s$$

Transversal vibrational
frequency depends on
whisker geometry and speed
of sound in its material

$$F \sim m \frac{\lambda}{\tau_b^2} \Rightarrow \tau_b \sim \frac{1}{\omega_b}$$


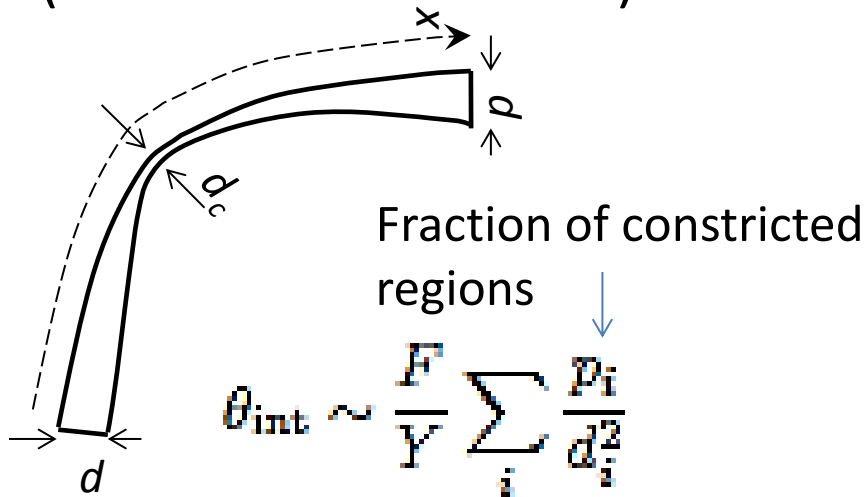
Newton's second law

Time of whisker
bending is reciprocal of
the above frequency

Typical times on the order of 1-100 ms consistent with observations

Some other applications

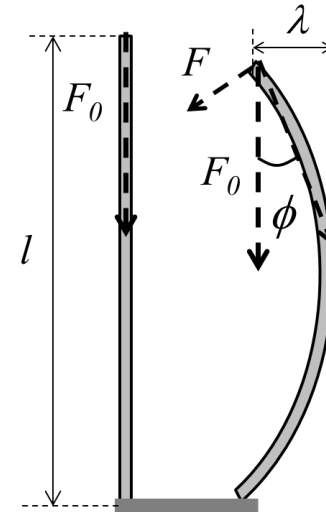
Whisker with constriction
(not sure how realistic)



Significant lowering of vibrational frequency

$$\omega_{be} \sim \omega_b \sqrt{\frac{l d_e^2}{d^3}} K_b \ll \omega_b,$$

Buckling instability



Critical compressive force

$$F_0 = F_{0c} \sim Y d^4 / l^2.$$

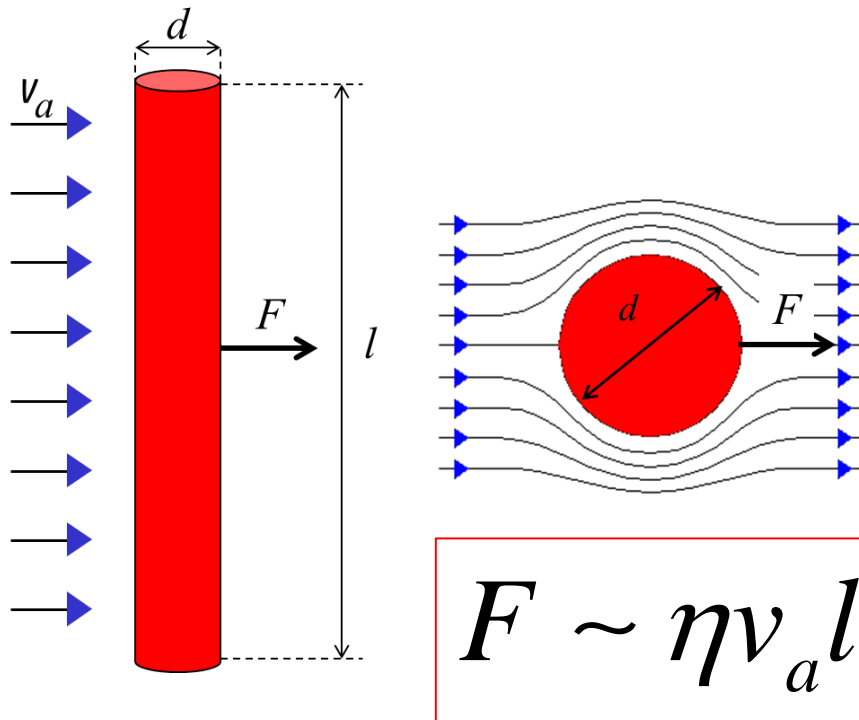
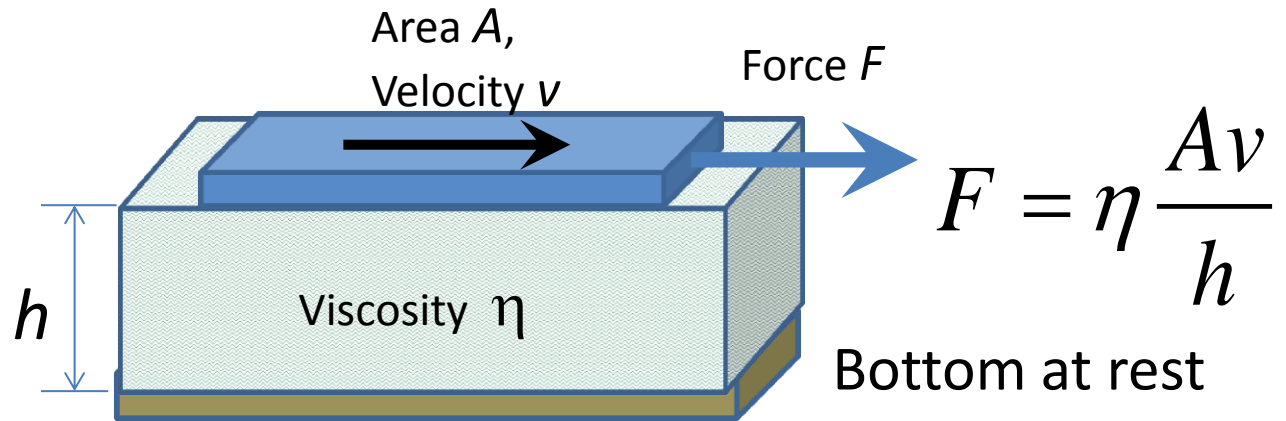
or critical whisker length

$$l_c = \sqrt{Y d^4 / F_0}.$$

above which buckling begins

Viscous drag on a whisker

Recalling the concept of viscosity



The only length across which the velocity field can change

$$h \rightarrow d$$

Side surface of a cylinder

$$A \rightarrow ld$$

6 mechanisms of whisker movements

Relevant material parameters Sn (Zn)

| Parameter | Value | Ref. |
|---|------------------------|-----------|
| density ρ g/cm ³ | 7.4 (7.1) ^a | 35 |
| Young's modulus Y GPa | 50 (108) | 35 |
| sound velocity, v_s , 10 ⁵ cm/s | 2.7 (3.9) | 35 |
| diameter, ^b d μ m | 0.1-20 | 2,3,36-38 |
| length, l | 10 μ m-25 mm | 2,3,36-38 |
| dynamic air viscosity, η g/cm-s | $2 \cdot 10^{-4}$ | 39 |
| kinematic air viscosity, η/ρ_a , cm ² /s | 0.15 | 39 |
| patch size, d_0 μ m | 0.1-10 | 7,8 |
| electric charge density, n e/cm ² | $10^{10} - 10^{12}$ | 7,8 |
| near surface field, E_0 V/cm, | $10^4 - 10^6$ | 7,8 |
| ion diffusion coefficient D_i , cm ² /s, | 10^{-16} | 40 |
| electron mobility μ_e , cm ² /V-s, | 1000 | 41 |

Structure of results

| Quantity | $d = 1\mu\text{m}$ $l = 1\text{mm}$ | $d = 1\mu\text{m}$ $l = 10\text{mm}$ | $d = 10\mu\text{m}$ $l = 1\text{mm}$ | $d = 10\mu\text{m}$ $l = 10\text{mm}$ |
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| $v_a \text{ cm/s,}$ Eq. (18) | $5 \cdot 10^2$ | 0.05 | $5 \cdot 10^6$ | $5 \cdot 10^3$ |
| Re^a | 500 | 0.5 | $5 \cdot 10^6$ | $5 \cdot 10^4$ |
| $\omega_b, \text{ s}^{-1}$ Eq. (5) | 3000 | 30 | 30,000 | 300 |
| $\Delta\phi, \text{ rad}$ Eq. (24) | 10^{-4} | $3 \cdot 10^{-4}$ | 10^{-6} | $3 \cdot 10^{-6}$ |
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| $\lambda/l, \text{ Eq. (27)}$ for $w = g$ | $2 \cdot 10^{-3}$ | 1 | $2 \cdot 10^{-5}$ | 0.02 |
| $\lambda/l, \text{ Eq. (29)}$ | 10^{-7} | 10^{-5} | 10^{-9} | 10^{-7} |
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^a $Re = l v_a \rho_a / \eta$ denotes the Reynolds number.

^bThe computed ratios $\lambda/l > 1$ are formal and must be replaced with 1.

^cFor the observation time $t = 1 \text{ s}$.

Quantity = estimated parameter that can be compared to observations

Neglecting continuous (~lognormal distributions) of whisker lengths and diameters, four reference points are taken:

Thin whiskers $d=1 \mu\text{m}$

Thick whiskers $d=10 \mu\text{m}$

Short whiskers $l=1 \text{ mm}$

Long whiskers $l=10 \text{ mm}$

1. Air flow

| Air flow cause | Air flow velocity (cm/s) ^a |
|----------------------------------|---------------------------------------|
| Room A/C | 400 |
| HVAC Vent | 220 |
| Person walking | 180 |
| Door opening | 120 |
| Expiratory air flow ^b | 100 |
| Diffuser vent | 25 |

^aData from Ref. 17, except the "expiratory" mode, for which the velocity was estimated assuming the average lungs expiratory volume¹⁸ of ~ 1 L and the lips expiration opening area ~ 1 cm² held for 10 s.

--

Yes, air flow can be a cause

Thin whiskers

Thick whiskers

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Air velocity
that produces
deflections
comparable to
whisker lengths.

Even thick
whiskers can be
moved under
strong enough
wind

$$F \sim \eta v_a l,$$

$$\lambda \sim Fl^3 / Yd^4$$

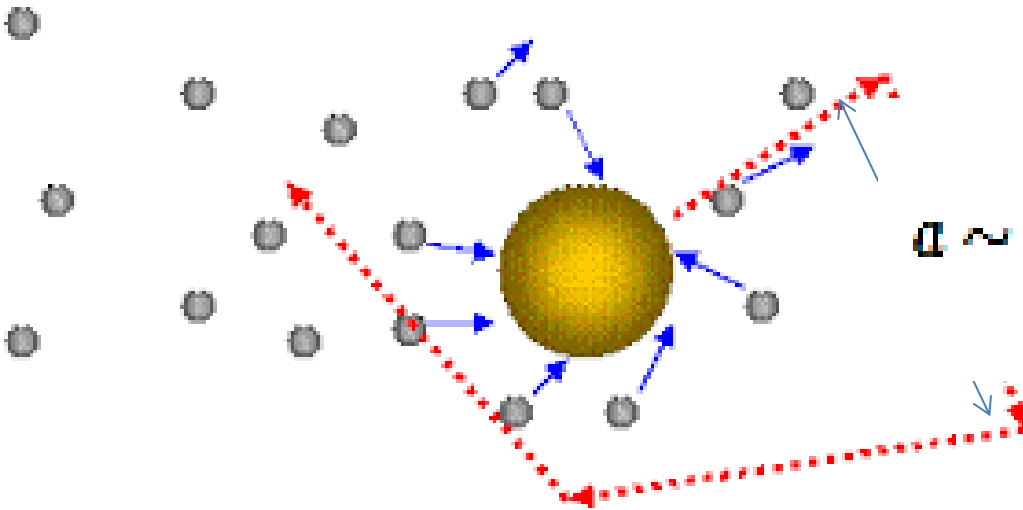
$$\lambda \sim l - \text{criterion}$$

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Brownian movements of 'free' particles

τ_η - coherency time

$$\eta v_T d \sim \frac{m v_T}{\tau_\eta}$$

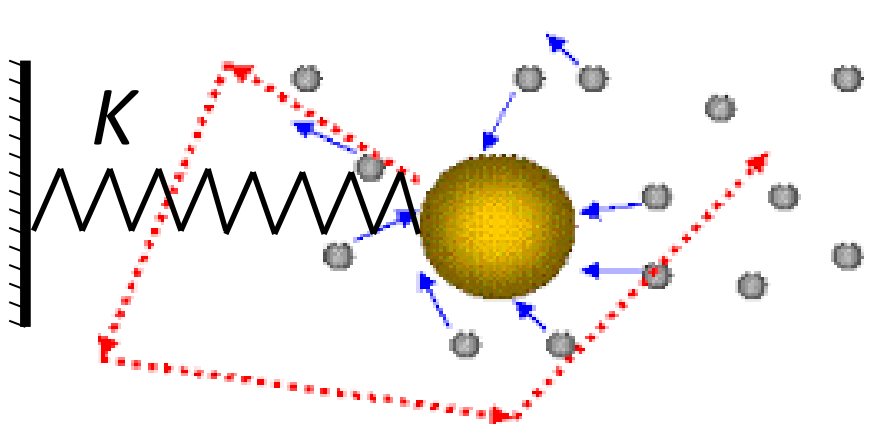


$$a \sim v_T \tau_\eta \sim \sqrt{\frac{k_B T}{m}} \tau_\eta \sim \frac{\sqrt{k_B T \rho d}}{\eta}$$

$$L \sim a \sqrt{\frac{t}{\tau_\eta}} \sim \sqrt{\frac{k_B T}{\eta d}} t \equiv \sqrt{D_d t} \quad \text{with} \quad D_d \sim \frac{k_B T}{\eta d}$$

Typical Brownian displacements are small: $\sim 10 \mu\text{m}$ per 1 s
(It took microscope to discover Brownian movements).

Modification: Brownian movements of a harmonic oscillator



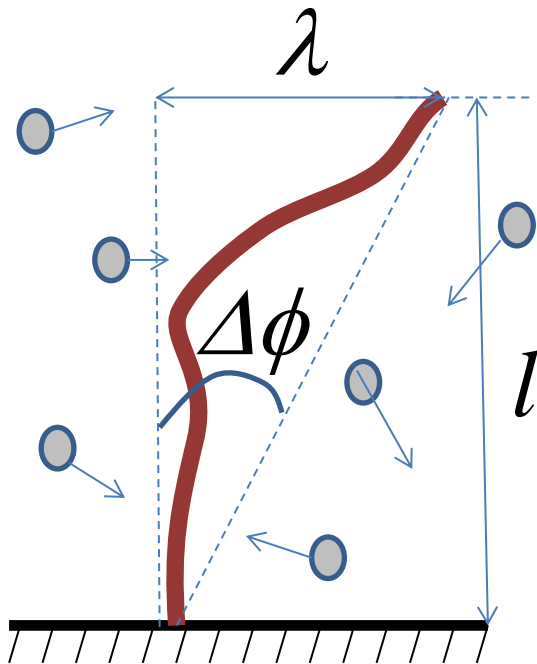
- The spring limits
- Brownian
- displacements to that of equipartition theorem

$$\frac{KL_{\max}^2}{2} = \frac{k_B T}{2}$$

The 'free' particle Brownian model works for $L < L_{\max}$
Brownian diffusion confined

Brownian movements of elastic beams

Limiting deflection is determined by bending elasticity K_b and thermal energy $k_B T$



$$\frac{K_b \lambda^2}{2} = \frac{k_B T}{2}$$

$$\Delta\phi \sim \sqrt{\frac{k_B T l}{Y d^4}}$$

$$t_{\Delta\phi} \sim \frac{\eta}{Y} \left(\frac{l}{d} \right)^4$$

This theory is consistent with the known results for bending fluctuations of long molecules

Brownian movements of metal whiskers is unlikely

Thin whiskers

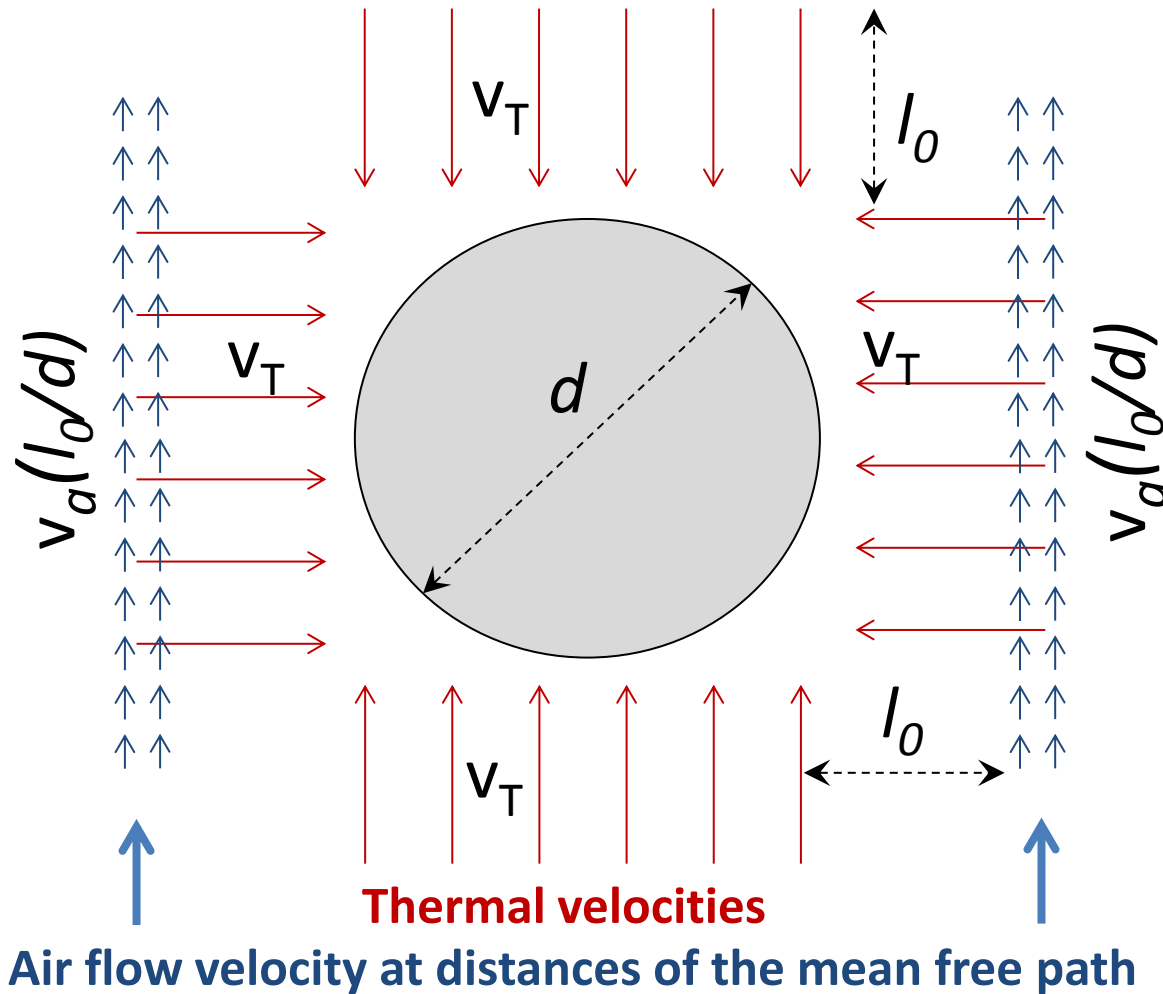
Thick whiskers

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Predicted angles are too small... maybe OK for severely constricted whiskers

Even though predicted time scales can be realistic

Brownian movements vs. viscous drag: what is so different?



Vertical air flow has a certain direction: momenta from both sides add

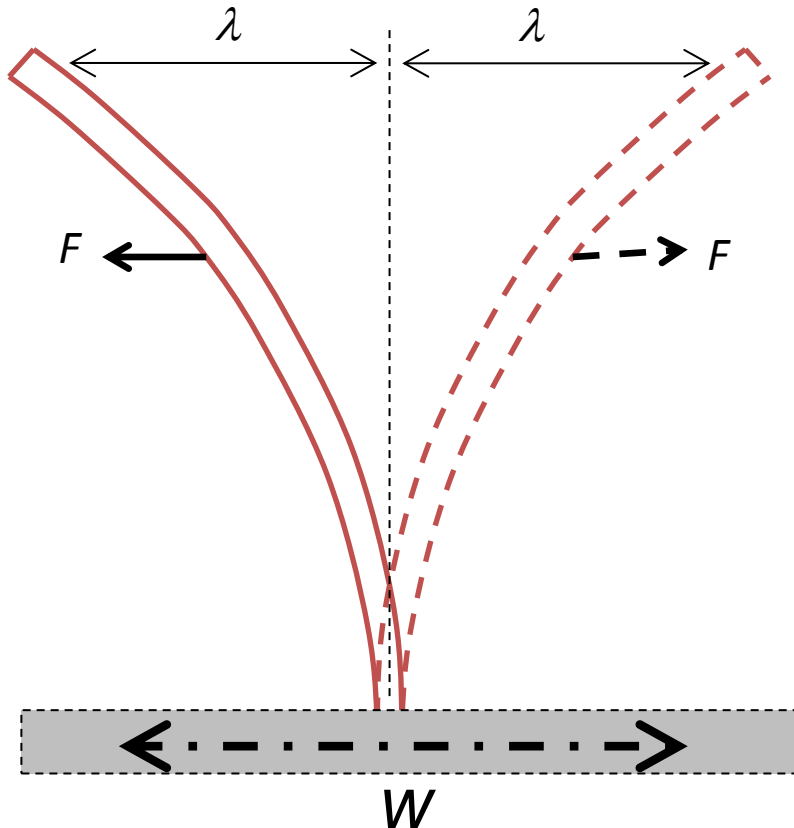
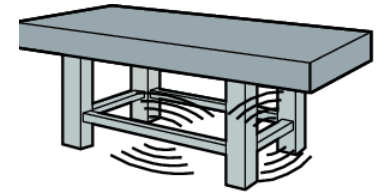
Brownian random momenta don't have preferential direction and mutually cancel

$$N v_a \frac{l_0}{d} > \sqrt{N} v_T$$

$$N \sim d^2 l_0 n \gg 1 \quad (!),$$

n – atomic concentration

External vibrations



Inertia force $F = -mw$

$$\text{deflection } \lambda = \frac{Fl^3}{Yd^4}$$

truck on a rural road $w \leq g$

standard lab $w < 0.01g$

External vibrations unlikely to move metal whiskers

Thin whiskers

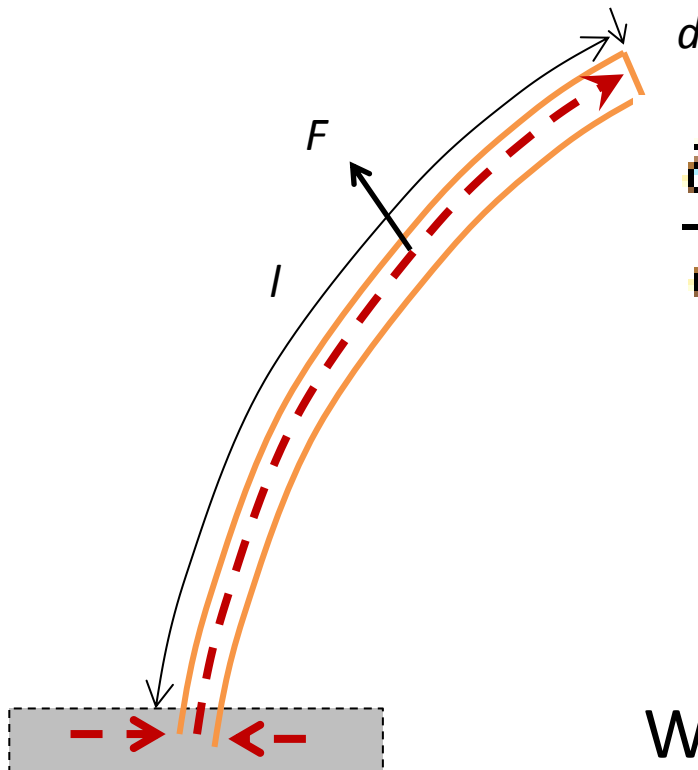
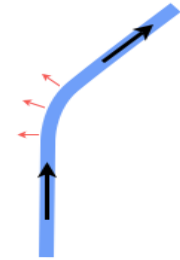
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Displacements
small, except
maybe
severely
constricted
whiskers

Garden hose instability



$$\frac{\partial p}{\partial t} = v_w \frac{\partial m}{\partial t} \sim v_w \rho d^2 \frac{\partial l}{\partial t} = \rho v_w^2 d^2$$

$$\frac{\lambda}{l} \sim \left(\frac{v_w}{v_s} \right)^2 \left(\frac{l}{d} \right)^2$$

Whisker growth velocity $v_w \sim 1 \text{ A/s}$ is so much smaller than the speed of sound $v_s \sim 10^5 \text{ cm/s}$ that this effect is strictly impossible

Garden hose instability irrelevant

| | Thin whiskers | | Thick whiskers | |
|---|--|---|---|--|
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Victor Karpov, Univ Toledo, April 2015

Random electric fields due to ionic diffusion



Diffusion can be represented as appearance of random dipoles

$$p^2 = e^2 D_i t,$$

$$(\delta E)^2 \sim \frac{(nl^2)e^2 D_i t}{l^6}$$

D_i – ion diffusion coefficient

n – ionic concentration per area

l – whisker length, t - time

Ionic diffusion is too slow

Thin whiskers

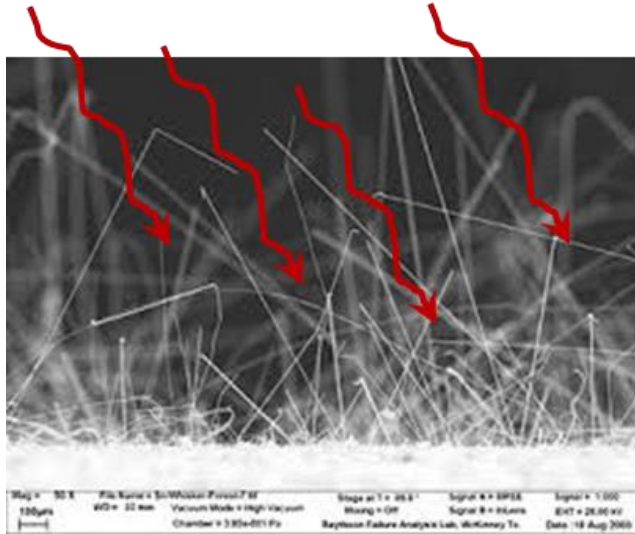
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| $\delta E/E$, Eq. (33) ^c with $\delta n/n = 1$ | 0.4 | 0.04 | 0.4 | 0.04 |



For $t = 1\text{s}$
Too slow
diffusion

Nonequilibrium electric charges

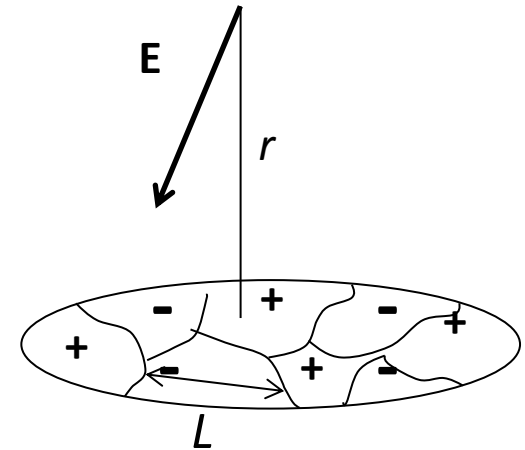
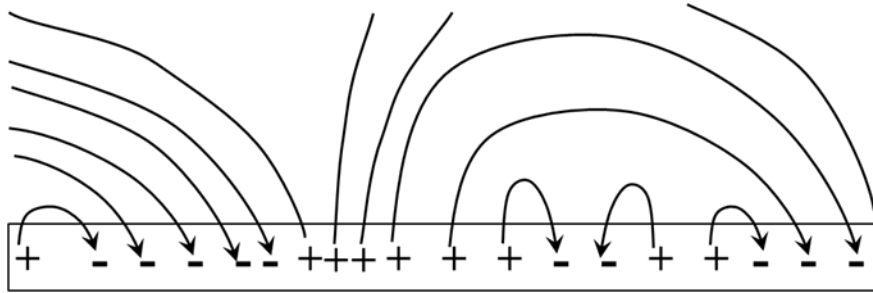


Light or ionizing radiation, or change in ambient ionization create nonequilibrium charge distribution altering electric fields on whiskers

This mechanism is built on the electrostatic theory of metal whiskers implying random charge patches on metal surfaces

Recalling the electrostatic theory (VK 2014)

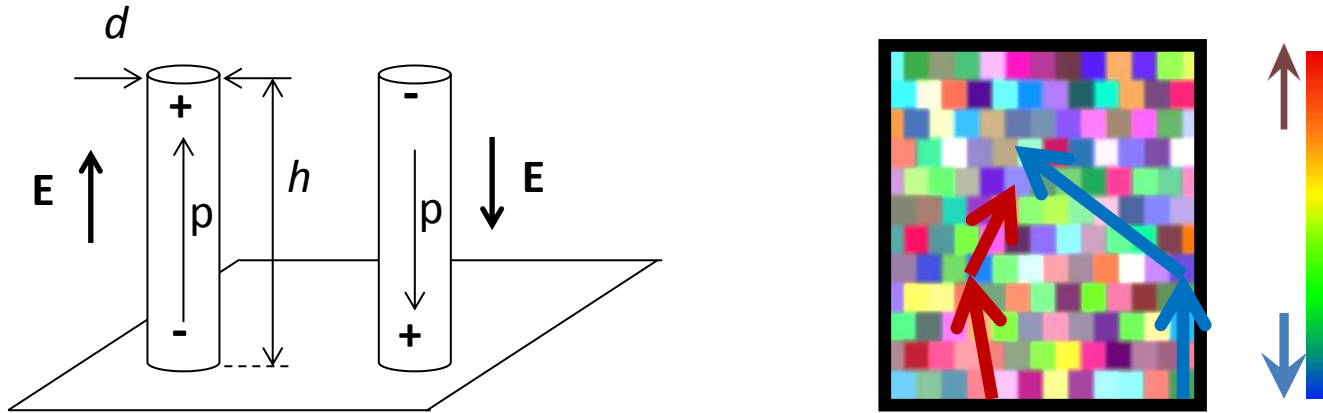
Charge patches



- Random charge patches due to imperfections: grain boundaries, oxides, ion contaminations, local deformations, dislocations, etc.
- Patch size $L \sim 0.1 - 10 \mu\text{m}$, field near the surface $E_0 \sim 10^4 - 10^6 \text{ V/cm}$
- At distances $r > L$ the field is random and decays, $|E| \sim |E_0| (L/r)$
- E-field theory explains the versatility of whisker triggers (GB, stresses, contaminations, humidity) and their random nature

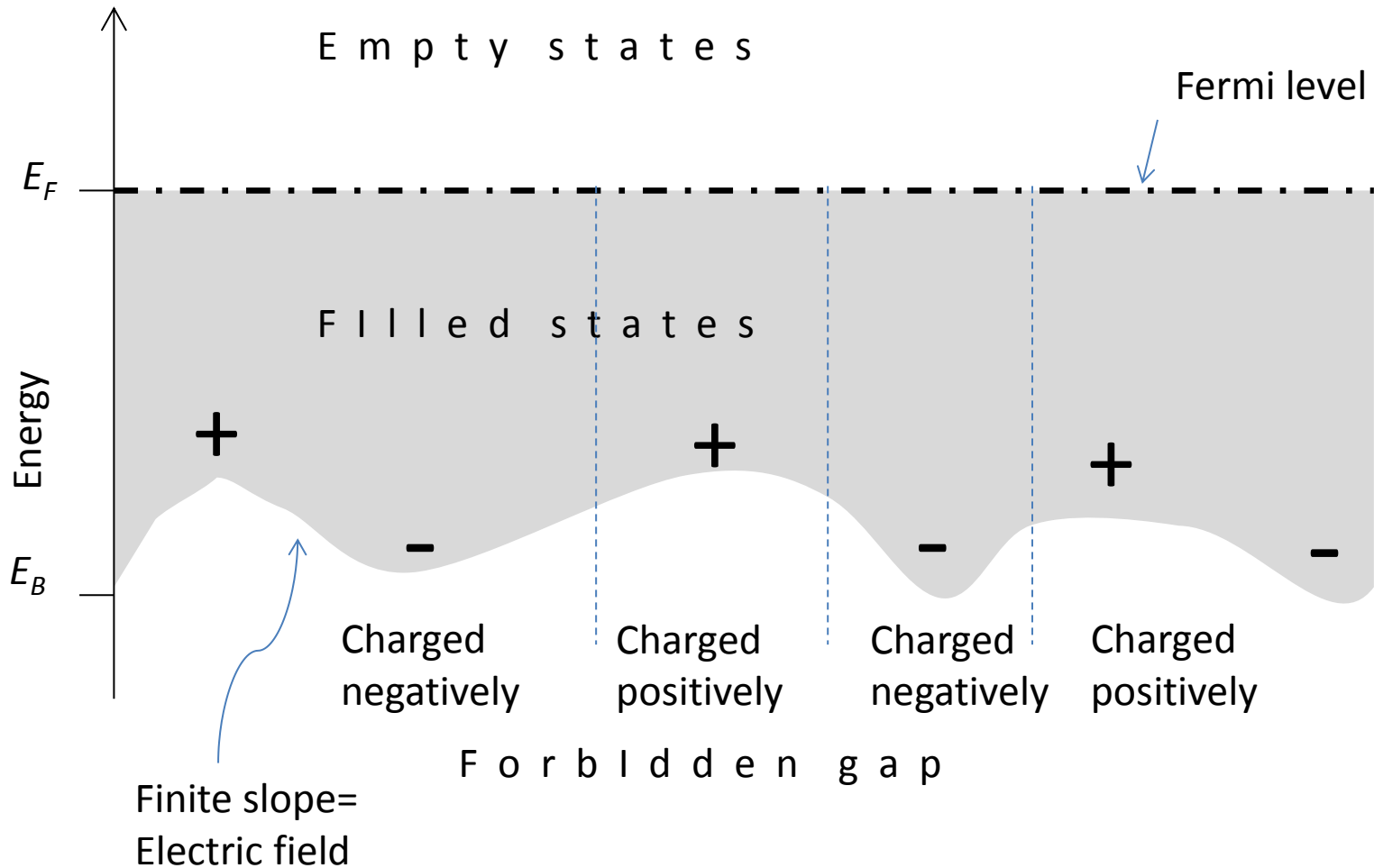
Recalling the electrostatic theory (cont)

Whisker development

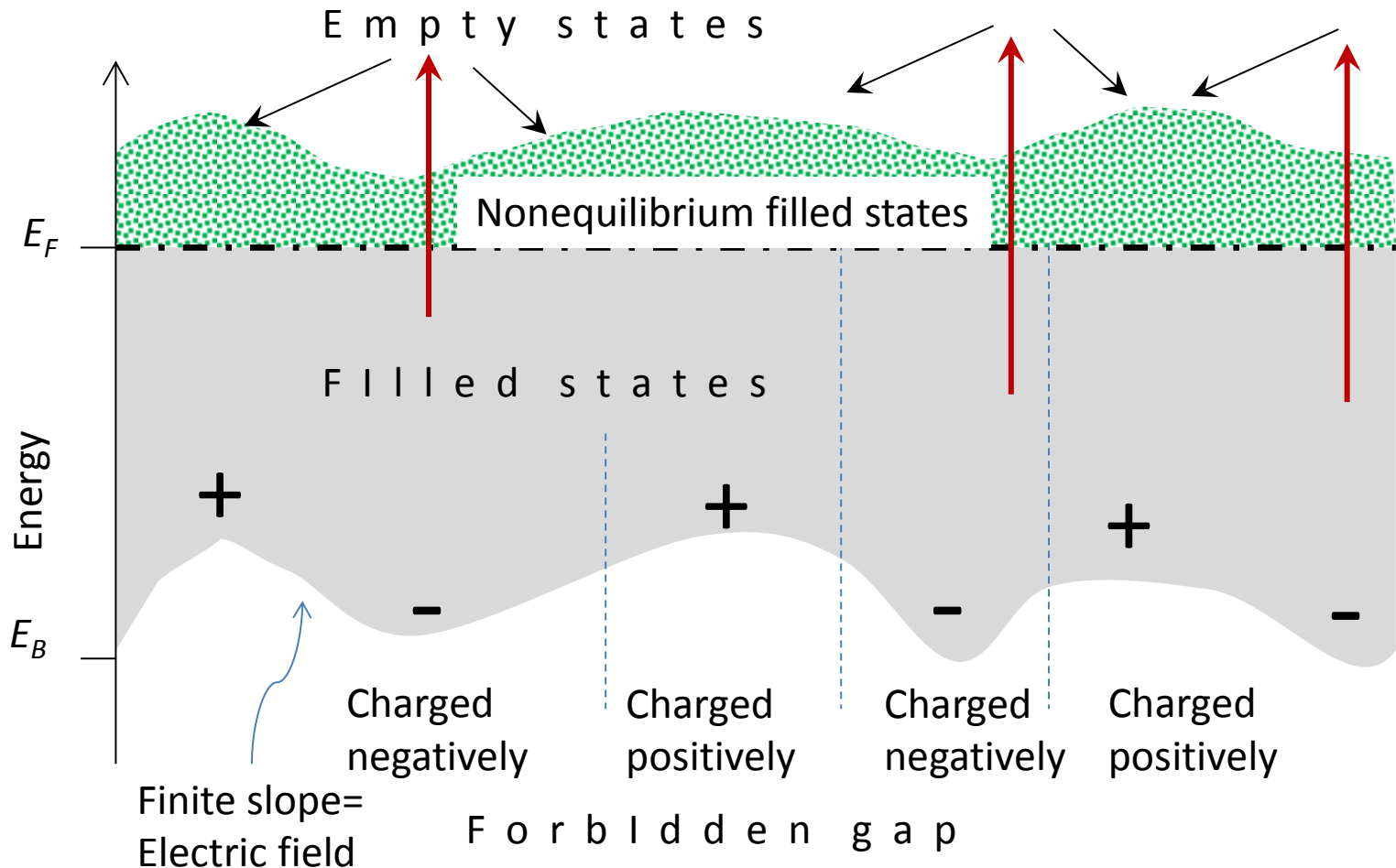


- Nucleation and growth of metal whiskers decreases system free energy because whiskers are polarized in the field
- High aspect ratios of up to $\sim 10,000$ naturally explained
- Growth kinetics: dormant period followed by constant growth rate
- Growth along pathways of not too different polarization (up or down)
- Random points of growth arrest determine the whisker length statistics: close to lognormal
- Quantitative estimates (for the first time) consistent with data

Equilibrium energy diagram



Nonequilibrium energy diagram



Photoinduced transitions level out the field
(electrons are moved to screen the existing fields)

Nonequilibrium charge features

- Previously unknown with metals
- Well known in many polycrystalline semiconductors
- Relaxation times up to months vary by many orders of magnitude between different materials and different samples, (trapping-detrapping, charging-recharging)
- Assuming the same true with metals could explain:
 - (a) possibility of ‘spontaneous’ whisker movements
 - (b) its unpredictable nature depending on sample recipe, light intensity and ambient, i. e.
 - (c) why some researchers observed ‘spontaneous’ whisker movements, while others didn’t

Nonequilibrium charge effect

Similar to ionic diffusion and using Einstein relation between electron diffusivity and mobility μ ,


$$\frac{\delta E}{E} \sim \frac{\delta n}{n} \sqrt{\frac{k_B T \mu_e t}{n e l^2 d_0^2}}.$$

This effect can be very strong even for nonequilibrium charge density comparable to the original charge density (moderate photoconductivity)

Nonequilibrium charges: estimate

| Quantity | $d = 1\mu\text{m}$ $l = 1\text{mm}$ | $d = 1\mu\text{m}$ $l = 10\text{mm}$ | $d = 10\mu\text{m}$ $l = 1\text{mm}$ | $d = 10\mu\text{m}$ $l = 10\text{mm}$ |
|--|--|---|---|--|
| v_{em}/s , Eq. (18) | $5 \cdot 10^2$ | 0.05 | $5 \cdot 10^4$ | $5 \cdot 10^3$ |
| Re^{B} | 500 | 0.5 | $5 \cdot 10^6$ | $5 \cdot 10^4$ |
| ω_p, s^{-1} Eq. (5) | 3000 | 30 | 30,000 | 300 |
| $\Delta\phi, \text{rad}$ Eq. (24) | 10^{-4} | $3 \cdot 10^{-4}$ | 10^{-6} | $3 \cdot 10^{-6}$ |
| $t_{\Delta\phi}, \text{s}$ Eq. (25) | $2 \cdot 10^{-4}$ | 2 | $2 \cdot 10^{-8}$ | $2 \cdot 10^{-4}$ |
| $\lambda/l, \text{Eq. (27)}$ for $\omega = \omega_p$ | $2 \cdot 10^{-2}$ | 1 | $2 \cdot 10^{-6}$ | 0.02 |
| $\lambda/l, \text{Eq. (29)}$ | 10^{-7} | 10^{-5} | 10^{-9} | 10^{-7} |
| $\lambda/l, \text{Eq. (30)},^{\text{B}}$ $\cos\theta \sim 1$ | $10^{-3} - 10$ | $0.1 - 10^3$ | $10^{-7} - 10^{-3}$ | $10 - 10^5$ |
| $\delta E/E$, Eq. (32) ^c | 10^{-9} | 10^{-10} | 10^{-9} | 10^{-10} |
| $\delta E/E$, Eq. (33) ^c with $\delta n/n = 1$ | 0.4 | 0.04 | 0.4 | 0.04 |

Effect
potentially
strong,
Not easily
predictable



Conclusions

- Air flow, even minute, can certainly be a cause of whisker movements
- Redistributing electrons in metal surface by light or other external agent can be a cause of whisker movements as well, although remains very sensitive to sample recipes and environment
- Metal whiskers can and should be systematically understood at a quantitative level – it is possible, and we do not even need computers for that